\*\*Summary of the Paper with Focus on AWE and Integration Method\*\*

### \*\*Asymptotic Waveform Evaluation (AWE)\*\*

AWE is a model-order reduction technique that employs \*\*Padé approximation\*\* to generate reduced-order rational approximations of high-order transfer functions. This significantly simplifies the analysis of large linear lumped/distributed networks. The method approximates the transfer function \( H(s) \) of a system using moments, which are coefficients of the Taylor series expansion of \( H(s) \) around a specific frequency (e.g., \( s = 0 \)):

\[

H(s) = m\_0 + m\_1s + m\_2s^2 + \dots

\]

AWE computes these moments and constructs a Padé approximation of the form:

\[

H\_{\text{AWE}}(s) = \frac{a\_0 + a\_1s + \dots + a\_qs^q}{1 + b\_1s + \dots + b\_ps^p}

\]

where \( q \) and \( p \) are the orders of the numerator and denominator polynomials, respectively. The coefficients \( a\_i, b\_i \) are determined by matching the moments of \( H\_{\text{AWE}}(s) \) to those of the original system. However, AWE faces limitations when applied to networks characterized by \*\*sampled data\*\* (e.g., scattering parameters), as derivatives required for moment calculations become inaccurate. Hybrid methods partition the frequency range and approximate each section with rational functions, but this introduces order-dependent errors.

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### \*\*Integration Method (Section V)\*\*

To avoid computationally expensive explicit convolution, the paper employs \*\*recursive convolution\*\* based on the pole-residue representation of the transfer function. For a system described in the Laplace domain as:

\[

Y(s) = \left( k\_\infty + \sum\_{i=1}^{\vartheta'} \frac{k\_i}{s + p\_i} \right) X(s),

\]

the time-domain response is derived using the inverse Laplace transform. Each pole-residue term \( \frac{k\_i}{s + p\_i} \) corresponds to a first-order differential equation:

\[

\frac{d}{dt} y\_i(t) + p\_i y\_i(t) = k\_i x(t). \tag{14}

\]

Assuming piecewise constant excitation \( x(t) \) over a time interval \( [t\_{n-1}, t\_n] \), the solution to (14) is:

\[

y\_i(t\_n) = k\_i \left( 1 - e^{-p\_i \Delta t} \right) x(t\_{n-1}) + e^{-p\_i \Delta t} y\_i(t\_{n-1}), \tag{15}

\]

where \( \Delta t = t\_n - t\_{n-1} \). The total response at \( t\_n \) is a superposition of all pole-residue contributions:

\[

y(t\_n) = k\_\infty x(t\_n) + \sum\_{i=1}^{\vartheta'} y\_i(t\_n). \tag{15}

\]

This recursive formulation is implemented as a \*\*Norton equivalent circuit\*\*:

- A constant conductance \( k\_\infty \),

- A time-dependent current source \( -\sum\_{i=1}^{\vartheta'} y\_i(t\_n) \), updated iteratively using past values.

The method reduces computational complexity from \( O(N^2) \) (explicit convolution) to \( O(N) \), where \( N \) is the number of time steps, by leveraging the exponential decay structure of the pole-residue terms.

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### \*\*Key Advantages\*\*

1. \*\*Efficiency\*\*: Recursive convolution avoids storing and convolving the entire history of the input.

2. \*\*Stability\*\*: The pole-residue model ensures stability by retaining only left-half-plane poles.

3. \*\*Compatibility\*\*: Direct integration into SPICE-like simulators using equivalent circuit stamps.

This approach eliminates the need for IFFT and band-limiting filters, enabling efficient transient simulation of high-speed interconnects with nonlinear terminations.